

Algebraic geometry 2

Exercise Sheet 12

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Exercise 1. Let $f : X \rightarrow Y$ be a morphism of schemes. Consider the respective diagonal morphism $\Delta : X \rightarrow X \times_Y X$. Show that Δ is a closed immersion if and only if the image of Δ is closed in $X \times_Y X$.

Exercise 2. Let A be a ring. Show that the morphism $\mathbb{P}_A^n \rightarrow \operatorname{Spec} A$ is separated.

Exercise 3. Let k be a field. Show that the affine line with doubled origin (constructed in Example 5.10) is not a separated scheme over k .

Exercise 4. (1) Let X and S' be schemes over S and let Y be a scheme over S' . Show that

$$(X \times_S S') \times_{S'} Y \simeq X \times_S Y.$$

(2) Let $i : Z \rightarrow X$ and $j : T \rightarrow Y$ be closed immersions over a scheme S . Show that $i \times j : Z \times_S T \rightarrow X \times_S Y$ is a closed immersion.

Hint: First consider the case $T = Y$ and use that closed immersions are stable under base change. Also use (1).